Chapter 6
Gravitation and Newton’s Synthesis
6-1 Newton’s Law of Universal Gravitation

If the force of gravity is being exerted on objects on Earth, what is the origin of that force?

Newton’s realization was that the force must come from the Earth.

He further realized that this force must be what keeps the Moon in its orbit.
The gravitational force on you is one-half of a third law pair: the Earth exerts a downward force on you, and you exert an upward force on the Earth.

When there is such a disparity in masses, the reaction force is undetectable, but for bodies more equal in mass it can be significant.
Therefore, the gravitational force must be proportional to both masses.

By observing planetary orbits, Newton also concluded that the gravitational force must decrease as the inverse of the square of the distance between the masses.

In its final form, the law of universal gravitation reads:

\[ F = G \frac{m_1 m_2}{r^2}, \]

where

\[ G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2. \]
The magnitude of the gravitational constant $G$ can be measured in the laboratory.

This is the Cavendish experiment.
Example 6-1: Can you attract another person gravitationally?

A 50-kg person and a 70-kg person are sitting on a bench close to each other. Estimate the magnitude of the gravitational force each exerts on the other.
**SOLUTION** We use Eq. 6–1, which gives

\[ F = G \frac{m_1 m_2}{r^2}, \]

\[
= \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(50 \text{ kg})(70 \text{ kg})}{(0.5 \text{ m})^2} \approx 10^{-6} \text{ N},
\]
Example 6-2: Spacecraft at $2r_E$.

What is the force of gravity acting on a 2000-kg spacecraft when it orbits two Earth radii from the Earth’s center (that is, a distance $r_E = 6380$ km above the Earth’s surface)? The mass of the Earth is $m_E = 5.98 \times 10^{24}$ kg.
EXAMPLE 6–2  Spacecraft at $2r_E$. What is the force of gravity acting on a 2000-kg spacecraft when it orbits two Earth radii from the Earth’s center (that is, a distance $r_E = 6380$ km above the Earth’s surface, Fig. 6–4)? The mass of the Earth is $m_E = 5.98 \times 10^{24}$ kg.

**APPROACH** We could plug all the numbers into Eq. 6–1, but there is a simpler approach. The spacecraft is twice as far from the Earth’s center as when it is at the surface of the Earth. Therefore, since the force of gravity decreases as the square of the distance (and $\frac{1}{2^2} = \frac{1}{4}$), the force of gravity on the satellite will be only one-fourth its weight at the Earth’s surface.

**SOLUTION** At the surface of the Earth, $F_G = mg$. At a distance from the Earth’s center of $2r_E$, $F_G$ is $\frac{1}{4}$ as great:

$$F_G = \frac{1}{4} mg = \frac{1}{4} (2000 \text{ kg})(9.80 \text{ m/s}^2) = 4900 \text{ N}.$$
Example 6-3: Force on the Moon.

Find the net force on the Moon ($m_M = 7.35 \times 10^{22}$ kg) due to the gravitational attraction of both the Earth ($m_E = 5.98 \times 10^{24}$ kg) and the Sun ($m_S = 1.99 \times 10^{30}$ kg), assuming they are at right angles to each other.
Example 6-3: Force on the Moon.

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**SOLUTION** The Earth is $3.84 \times 10^5 \text{ km} = 3.84 \times 10^8 \text{ m}$ from the Moon, so $F_{ME}$ (the gravitational force on the Moon due to the Earth) is

$$F_{ME} = \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(7.35 \times 10^{22} \text{ kg})(5.98 \times 10^{24} \text{ kg})}{(3.84 \times 10^8 \text{ m})^2} = 1.99 \times 10^{20} \text{ N.}$$

The Sun is $1.50 \times 10^8 \text{ km}$ from the Earth and the Moon, so $F_{MS}$ (the gravitational force on the Moon due to the Sun) is

$$F_{MS} = \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(7.35 \times 10^{22} \text{ kg})(1.99 \times 10^{30} \text{ kg})}{(1.50 \times 10^{11} \text{ m})^2} = 4.34 \times 10^{20} \text{ N.}$$

The two forces act at right angles in the case we are considering (Fig. 6–5), so we can apply the Pythagorean theorem to find the magnitude of the total force:

$$F = \sqrt{(1.99 \times 10^{20} \text{ N})^2 + (4.34 \times 10^{20} \text{ N})^2} = 4.77 \times 10^{20} \text{ N.}$$

The force acts at an angle $\theta$ (Fig. 6–5) given by $\theta = \tan^{-1}(1.99/4.34) = 24.6^\circ$. 

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6-2 Vector Form of Newton’s Universal Gravitation

In vector form,

$$\vec{F}_{12} = -G \frac{m_1 m_2}{r_{21}^2} \hat{r}_{21}. \quad (6-2)$$

This figure gives the directions of the displacement and force vectors.
6-2 Vector Form of Newton’s Universal Gravitation

If there are many particles, the total force is the vector sum of the individual forces:

\[ \vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \cdots + \vec{F}_{1n} = \sum_{i=2}^{n} \vec{F}_{1i}. \]
Now we can relate the gravitational constant to the local acceleration of gravity. We know that, on the surface of the Earth:

\[ mg = G \frac{mm}{r^2} \]

Solving for \( g \) gives:

\[ g = G \frac{m}{r^2} \]

Now, knowing \( g \) and the radius of the Earth, the mass of the Earth can be calculated:

\[ m_E = \frac{gr^2}{G} - 5.98 \times 10^{24} \text{ kg} \]
Example 6-4: Gravity on Everest.

Estimate the effective value of $g$ on the top of Mt. Everest, 8850 m (29,035 ft) above sea level. That is, what is the acceleration due to gravity of objects allowed to fall freely at this altitude?
Example 6-4: Gravity on Everest.

Estimate the effective value of \( g \) on the top of Mt. Everest, 8850 m (29,035 ft) above sea level. That is, what is the acceleration due to gravity of objects allowed to fall freely at this altitude?

**SOLUTION** We use Eq. 6–4, with \( r_E \) replaced by \( r = 6380 \text{ km} + 8.9 \text{ km} = 6389 \text{ km} = 6.389 \times 10^6 \text{ m} \):

\[
g = G \frac{m_E}{r^2} = \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}{(6.389 \times 10^6 \text{ m})^2}
\]

\[
= 9.77 \text{ m/s}^2,
\]

which is a reduction of about 3 parts in a thousand (0.3%).

**NOTE** This is an estimate because, among other things, we ignored the mass accumulated under the mountaintop.
The acceleration due to gravity varies over the Earth’s surface due to altitude, local geology, and the shape of the Earth, which is not quite spherical.

### TABLE 6–1

<table>
<thead>
<tr>
<th>Location</th>
<th>Elevation (m)</th>
<th>g (m/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>New York</td>
<td>0</td>
<td>9.803</td>
</tr>
<tr>
<td>San Francisco</td>
<td>0</td>
<td>9.800</td>
</tr>
<tr>
<td>Denver</td>
<td>1650</td>
<td>9.796</td>
</tr>
<tr>
<td>Pikes Peak</td>
<td>4300</td>
<td>9.789</td>
</tr>
<tr>
<td>Sydney, Australia</td>
<td>0</td>
<td>9.798</td>
</tr>
<tr>
<td>Equator</td>
<td>0</td>
<td>9.780</td>
</tr>
<tr>
<td>North Pole (calculated)</td>
<td>0</td>
<td>9.832</td>
</tr>
</tbody>
</table>
6-4 Satellites and “Weightlessness”

Satellites are routinely put into orbit around the Earth. The **tangential speed** must be high enough so that the satellite does not return to Earth, but not so high that it escapes Earth’s gravity altogether.
The satellite is kept in **orbit by its speed**—it is continually falling, but the Earth curves from underneath it.
6-4 Satellites and “Weightlessness”

Example 6-6: Geosynchronous satellite.

A geosynchronous satellite is one that stays above the same point on the Earth, which is possible only if it is above a point on the equator. Such satellites are used for TV and radio transmission, for weather forecasting, and as communication relays. Determine (a) the height above the Earth’s surface such a satellite must orbit, and (b) such a satellite’s speed. (c) Compare to the speed of a satellite orbiting 200 km above Earth’s surface.
SOLUTION

(a) We apply Eq. 6–5, assuming the satellite moves in a circle:

\[ G \frac{m_{\text{Sat}} m_E}{r^2} = m_{\text{Sat}} \frac{v^2}{r}. \]

This equation has two unknowns, \( r \) and \( v \). But the satellite revolves around the Earth with the same period that the Earth rotates on its axis, namely once in 24 hours. Thus the speed of the satellite must be

\[ v = \frac{2\pi r}{T}, \]

where \( T = 1 \text{ day} = (24 \text{ h})(3600 \text{ s/h}) = 86,400 \text{ s} \). We substitute this into the “satellite equation” above and obtain (after canceling \( m_{\text{Sat}} \) on both sides)

\[ G \frac{m_E}{r^2} = \frac{(2\pi r)^2}{rT^2}. \]

After cancelling an \( r \), we can solve for \( r^2 \):

\[ r^3 = \frac{G m_E T^2}{4\pi^2} = \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})(86,400 \text{ s})^2}{4\pi^2} \]

\[ = 7.54 \times 10^{22} \text{ m}^3. \]

We take the cube root and find

\[ r = 4.23 \times 10^7 \text{ m}, \]
(b) We solve for $v$ in the satellite equation, Eq. 6–5:

$$
v = \sqrt{\frac{Gm_{E}}{r}} = \sqrt{\frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}{(4.23 \times 10^7 \text{ m})}} = 3070 \text{ m/s.}
$$

We get the same result if we use $v = 2\pi r/T$.

(c) The equation in part (b) for $v$ shows $v \propto \sqrt{1/r}$. So for $r = r_E + h = 6380 \text{ km} + 200 \text{ km} = 6580 \text{ km}$, we get

$$
v' = v \sqrt{\frac{r}{r'}} = (3070 \text{ m/s}) \sqrt{\frac{(42,300 \text{ km})}{(6580 \text{ km})}} = 7780 \text{ m/s}.
$$
6-4 Satellites and “Weightlessness”

Objects in orbit are said to experience weightlessness. They do have a gravitational force acting on them, though! The satellite and all its contents are in free fall, so there is no normal force. This is what leads to the experience of weightlessness.

\[ \vec{a} = 0 \]
\[ \vec{a} = \frac{1}{2} \vec{g} \text{ (up)} \]
\[ \vec{a} = \vec{g} \text{ (down)} \]
6-4 Satellites and “Weightlessness”

More properly, this effect is called apparent weightlessness, because the gravitational force still exists. It can be experienced on Earth as well, but only briefly:
8-7 Gravitational Potential Energy and Escape Velocity

Far from the surface of the Earth, the force of gravity is not constant:

\[ \vec{F} = -G \frac{mM_E}{r^2} \hat{r}. \]

The work done on an object moving in the Earth’s gravitational field is given by:

\[ W = \int_1^2 \vec{F} \cdot d\vec{\ell} = -GmM_E \int_1^2 \frac{\hat{r} \cdot d\vec{\ell}}{r^2}. \]
Gravitational Potential Energy

From the definition of potential energy

$$U = -\int_{\text{ref}}^{r} \mathbf{F} \cdot d\mathbf{r}$$

and the law of gravitation

$$\mathbf{F} = -\frac{GMm}{r^2} \mathbf{1}_r$$

with the choice of the zero of potential energy at infinite distance where the force approaches zero, the gravitational potential energy is the work done to bring an object from infinity to radius $r$:

$$U(r) = -\int_{\infty}^{r} \frac{GMm}{r'^2} dr' = -\frac{GMm}{r}$$

$\mathbf{1}_r$ represents a unit vector in the outward radial direction.
Gravitational Potential Energy
From the work done against the gravity force in bringing a mass in from infinity where the potential energy is assigned the value zero, the expression for gravitational potential energy is

\[ U = \frac{-GMm}{r} \]

This expression is useful for the calculation of escape velocity, energy to remove from orbit, etc. However, for objects near the earth the acceleration of gravity \( g \) can be considered to be approximately constant and the expression for potential energy relative to the Earth’s surface becomes

\[ U = mgh \]

where \( h \) is the height above the surface and \( g \) is the surface value of the acceleration of gravity.
Escape Velocity

If the kinetic energy of an object launched from the Earth were equal in magnitude to the potential energy, then in the absence of friction resistance it could escape from the Earth.

\[ v_{\text{escape}} = 11.2 \text{ km/s} \]

\[ \frac{1}{2}mv^2 = \frac{GMm}{r} \]

\[ v_{\text{escape}} = \sqrt{\frac{2GM}{r}} \]
Orbit Velocity and Escape Velocity

If the kinetic energy of an object $m_1$ launched from the a planet of mass $M_2$ were equal in magnitude to the potential energy, then in the absence of friction resistance it could escape from the planet. The escape velocity is given by

$$\frac{Gm_1M_2}{R^2} = \frac{m_1v^2}{R}$$

$$v_{\text{orbit}} = \sqrt{\frac{GM_2}{R}}$$

to find the orbit velocity for a circular orbit, you can set the gravitational force equal to the required centripetal force.

$$\frac{Gm_1M_2}{R^2} = \frac{m_1v^2}{R}$$

$$v_{\text{orbit}} = \sqrt{\frac{GM_2}{R}}$$

Note that the orbit velocity and the escape velocity from that radius are related by

$$v_{\text{escape}} = \sqrt{2} v_{\text{orbit}}$$
8-7 Gravitational Potential Energy and Escape Velocity

Solving the integral gives:

\[ W = \frac{GmM_E}{r_2} - \frac{GmM_E}{r_1}. \]

Because the value of the integral depends only on the end points, the gravitational force is conservative and we can define gravitational potential energy:

\[ U(r) = -\frac{GmM_E}{r}. \]
8-7 Gravitational Potential Energy and Escape Velocity

Example 8-12: Package dropped from high-speed rocket.

A box of empty film canisters is allowed to fall from a rocket traveling outward from Earth at a speed of 1800 m/s when 1600 km above the Earth’s surface. The package eventually falls to the Earth. Estimate its speed just before impact. Ignore air resistance.
Example 8-12: Package dropped from high-speed rocket.

A box of empty film canisters is allowed to fall from a rocket traveling outward from Earth at a speed of 1800 m/s when 1600 km above the Earth's surface. The package eventually falls to the Earth. Estimate its speed just before impact. Ignore air resistance.

**SOLUTION** Conservation of energy in this case is expressed by Eq. 8–18:

$$\frac{1}{2}mv_1^2 - G\frac{mM_E}{r_1} = \frac{1}{2}mv_2^2 - G\frac{mM_E}{r_2}$$

where $v_1 = 1.80 \times 10^3$ m/s, $r_1 = (1.60 \times 10^6$ m) + (6.38 $\times 10^6$ m) = 7.98 $\times 10^6$ m, and $r_2 = 6.38 \times 10^6$ m (the radius of the Earth). We solve for $v_2$:

$$v_2 = \sqrt{v_1^2 - 2GM_E\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

$$= \sqrt{(1.80 \times 10^3 \text{ m/s})^2 - 2(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})} \times \left(\frac{1}{7.98 \times 10^6 \text{ m}} - \frac{1}{6.38 \times 10^6 \text{ m}}\right)$$

$$= 5320 \text{ m/s.}$$
8-7 Gravitational Potential Energy and Escape Velocity

If an object’s initial kinetic energy is equal to the potential energy at the Earth’s surface, its total energy will be zero. The velocity at which this is true is called the escape velocity; for Earth:

$$v_{\text{esc}} = \sqrt{\frac{2GM_{\text{E}}}{r_{\text{E}}}} = 1.12 \times 10^4 \text{ m/s}.$$
Example 8-13: Escaping the Earth or the Moon.

(a) Compare the escape velocities of a rocket from the Earth and from the Moon.

(b) Compare the energies required to launch the rockets. For the Moon, $M_M = 7.35 \times 10^{22} \text{ kg}$ and $r_M = 1.74 \times 10^6 \text{ m}$, and for Earth, $M_E = 5.98 \times 10^{24} \text{ kg}$ and $r_E = 6.38 \times 10^6 \text{ m}$. 
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**SOLUTION** (a) Using Eq. 8–19, the ratio of the escape velocities is

\[
v_{\text{esc}} = \sqrt{\frac{2GM}{r}} = 1.12 \times 10^4 \text{ m/s}
\]

\[
\frac{v_{\text{esc}}(\text{Earth})}{v_{\text{esc}}(\text{Moon})} = \sqrt{\frac{M_E}{M_M}} \frac{r_M}{r_E} = 4.7.
\]

To escape Earth requires a speed 4.7 times that required to escape the Moon. (b) The fuel that must be burned provides energy proportional to \( v^2 \) \((K = \frac{1}{2}mv^2)\); so to launch a rocket to escape Earth requires \((4.7)^2 = 22\) times as much energy as to escape from the Moon.
Determine the force of gravitational attraction between the Earth and a 75 kg student if the student is standing at sea level, a distance of $6.4 \times 10^6$ m from the Earth’s center. [Mass of Earth $= 5.98 \times 10^{24}$ kg]

**Solution**

The gravitational attraction force

$$F_G = \frac{G m_{Earth} m_{student}}{r^2}$$

$$= \frac{(6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2})(5.98 \times 10^{24} \text{ kg})(75 \text{ kg})}{(6.4 \times 10^6 \text{ m})^2}$$

$$= 7.30 \times 10^2 \text{ N}$$
Find the gravitational acceleration $g$ on planet Venus, assuming the planet to be a sphere of radius 6052 km. [Mass of Venus = $4.87 \times 10^{24}$ kg]

**Solution**

Let $m_{Venus}$ be the mass of Venus, and $m$ the mass of an object on Venus’s surface. From the gravitational force and the weight of the object

$$\frac{G m_{Venus}}{r_{Venus}^2} m = mg$$

$$g = \frac{G m_{Venus}}{r_{Venus}^2}$$

$$= \frac{(6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2})(4.87 \times 10^{24} \text{ kg})}{(6.052 \times 10^6 \text{ m})^2}$$

$$= 8.87 \text{ m s}^{-2}$$
What is the acceleration due to gravity on top of Mount Kinabalu, the highest peak in Malaysia? The peak is about 4.1 km above the sea level.

[Radius of Earth, \( r_{\text{Earth}} = 6380 \) km, Mass of Earth, \( m_{\text{Earth}} = 5.98 \times 10^{24} \) kg]

**Solution**

\[
a_g = \frac{GM_{\text{Earth}}}{(R_{\text{Earth}} + h)^2}
\]

\[
= \frac{(6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2})(5.98 \times 10^{24} \text{ kg})}{(6.38 \times 10^6 \text{ m} + 4.1 \times 10^3 \text{ m})^2}
\]

\[
= 9.79 \text{ m s}^{-2}
\]
An 85 kg woman weighs 834 N on Earth’s surface. How far above the surface would she have to go to “lose” 40% of her body weight?

**Solution**

The 40% weight reduction is caused by a 40% reduction in gravitational acceleration.

From $a_g = \frac{GM_{Earth}}{(R_{Earth} + h)^2}$

$$h = \sqrt{\frac{GM_{Earth}}{a_g}} - R_{Earth}$$

$$= \sqrt{\frac{(6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2})(5.98 \times 10^{24} \text{ kg})}{0.60(9.81 \text{ m s}^{-2})}} - 6.38 \times 10^6 \text{ m}$$

$$= 1.85 \times 10^6 \text{ m}$$
Calculate the escape velocity of a rocket leaving planet Saturn.
[Radius of planet Saturn = 60268 km, Mass of planet Saturn = 5.69 × 10^{26} kg]

Solution

Escape velocity, \( v_{esc} \) is given by the formula

\[
v_{esc} = \sqrt{\frac{2Gm_{Saturn}}{r_{Saturn}}}
\]

\[
= \sqrt{\frac{2(6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2})(5.69 \times 10^{26} \text{ kg})}{60.3 \times 10^6 \text{ m}}}
\]

\[
= 3.55 \times 10^4 \text{ m s}^{-1}
\]